

Lecture 2

The Production Process

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Economic Growth and
Economic Fluctuations

Purpose of this Lecture

This lecture is **NOT** intended to give you a comprehensive understanding of firm behavior.

If you're interested in how economists model firm behavior, then take my course in Microeconomics.

The purpose is to provide you with a basic understanding of:

- marginal product of labor and marginal product of capital
 - why firms hire labor until the wage rate is equal to price times the marginal product of labor, i.e. $w = p \cdot MPL$
 - why firms hire capital until the rental rate on capital is equal to price times the marginal product of capital, i.e. $r = p \cdot MPK$
- assumption that firms make zero economic profit in the long-run if they face **constant returns to scale**

Intro to Firm Behavior

- **production** – process by which inputs are combined, transformed, and turned into outputs
- **firm** – person or a group of people that produce a good or service to meet a perceived demand
- we'll assume that firms' goal is to **maximize profit**

Perfect Competition

- many firms, each small relative to overall size of the industry, producing homogenous (virtually identical) products
- no firm is large enough to have any control over price
- new competitors can freely enter and exit the market

Competitive Firms are Price Takers

- firms have no control over price
- price is determined by the market

Firms' Basic Decisions

1. How much of each input to demand
2. Which production technology to use
3. How much supply

Short-Run vs. Long-Run

In the **short-run**, two conditions hold:

1. firm is operating under a fixed scale of production – i.e. at least one input is held fixed (ex. it may be optimal for a firm to buy new machinery, but it can't do so overnight)
2. firms can neither enter nor exit an industry

In the **long-run**:

- there are no fixed factors of production, so firms can freely increase or decrease scale operation
- new firms can enter and existing firms can exit the industry

Profit-Maximization

(economic) **profit = total revenue – total (economic) cost**

total revenue – amount received from the sale of the product
(price times number of goods sold)

total (economic) cost – the total of:

1. **out of pocket costs** (ex. prices paid to each input)
2. **opportunity costs:**
 - a. **normal rate of return on capital and**
 - b. **opportunity cost of each factor of production** – ex. if an employee in my firm could earn \$30,000 if he/she worked for another firm, then I'd have to pay him/her at least \$30,000, otherwise he/she would leave. (In reality, you might work in your parents' firm and not be paid. In such a case, their firm's accounting profit would be higher than their firm's economic profit).

normal rate of return on capital – rate of return that is just sufficient to keep investors satisfied (ex. real interest rate on corporate bonds)

- nearly the same as the real interest rate on risk-free government bonds for relatively risk-free firms
- higher for relatively more risky firms

Production Process

optimal method of production minimizes cost

production technology – relationship betw/n inputs & outputs

- labor-intensive technology relies heavily on labor instead of capital
- capital-intensive technology relies heavily on capital instead of labor

production function – units of **total product** as func. of units of inputs

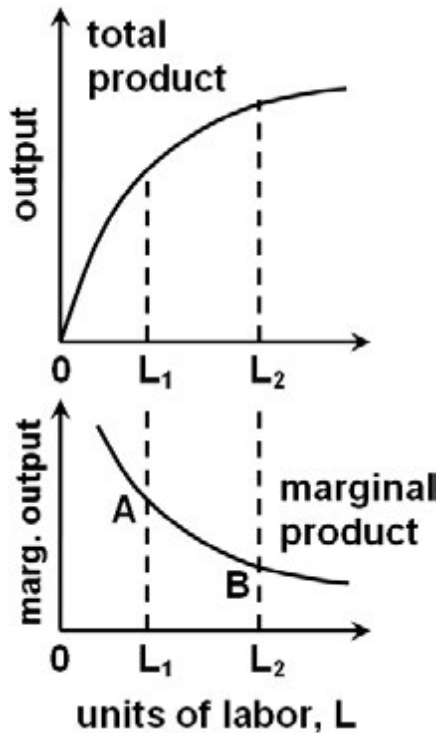
average product – average amount produced by each unit of a variable factor of production (input)

$$\text{avg. product of labor} = \frac{\text{total product}}{\text{total units of labor used}}$$

marginal product – additional output produced by adding one more unit of a variable factor of production (input), *ceteris paribus*

$$\text{marg. product of labor} = \frac{\Delta \text{ total product}}{\Delta \text{ units of labor used}}$$

Total and Marginal Product



diminishing marginal returns – when additional units of a variable input are added to fixed inputs, the marginal product of the variable input declines

marginal product is the slope of the total product function

at labor input L_1 , the slope of the total product function is relatively higher than it is at L_2 , so the marginal product of labor is higher at **point A** than it is at **point B**

in the case depicted, the marginal product of labor diminishes over entire range of labor input

Production with Two Inputs

Inputs often work together and are complementary.

- Ex. cooks (Labor) and grills (Capital)
- If you hire more cooks, but don't add any more grills, the marginal product of labor falls (too many cooks in the kitchen)
- If you hire (rent) more grills, but don't add any more cooks, the marginal product of capital falls (grills sit idle).

Given the technologies available, a profit-maximizing firm:

- hires labor up to the point where the wage equals the price times the marginal product of labor (MPL)
- hires capital up to the point where the rental rate on capital equals the price times the marginal product of capital (MPK)

$$\text{Profits } (\Pi) = \text{Total Revenue} - \text{Total Cost}$$

$$\Pi = p_X X(K, L) - wL - rK$$

at profit maximum:

$$w = p_X \text{MPL}$$

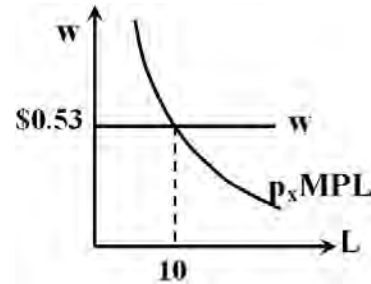
$$r = p_X \text{MPK}$$

Profit Maximization with Two Inputs

- In a perfectly competitive industry:
 - firms are price takers in both input output markets,
 - firms cannot affect the price of their product, nor can they affect the price of inputs (wages, rental rate on capital).
- Assume that a firm produces X with capital and labor and that its production function is given by:

$$X = K^{2/3} * L^{1/3}$$

- Assume also that:
 - price of the firm's output is \$1
 - wage rate and rental rate are both \$0.53



- So its profits are given by:

$$\begin{aligned} \Pi &= p_x X - r * K - w * L \\ &= \$1 * K^{2/3} * L^{1/3} - \$0.53 * K - \$0.53 * L \end{aligned}$$

- and firm should hire labor until $w = p_x \text{MPL}$

Profit Maximization with Two Inputs

- If the firm has 20 units of capital on hand (this is the short-run):

- how much labor should it hire?
- how much should it produce?
- how much profit will it make?

Output of X	Capital	Labor	MPL
14.74	20	8	0.61
15.33	20	9	0.57
15.87	20	10	0.53
16.39	20	11	0.50
16.87	20	12	0.47

- In the case depicted, the firm:

- would hire 10 labor units
- would produce 15.87 units of X
- would make **zero profit** ... (I'm foreshadowing a little here)

$$\Pi^* = \$1 * 15.87 - \$0.53 * 20 - \$0.53 * 10 = \$0$$

- Hiring more labor or less labor would lower profit:

$$\Pi_8 = \$1 * 14.74 - \$0.53 * 20 - \$0.53 * 8 = -\$0.14$$

$$\Pi_9 = \$1 * 15.33 - \$0.53 * 20 - \$0.53 * 9 = -\$0.07$$

$$\Pi_{11} = \$1 * 16.39 - \$0.53 * 20 - \$0.53 * 11 = -\$0.03$$

$$\Pi_{12} = \$1 * 16.87 - \$0.53 * 20 - \$0.53 * 12 = -\$0.06$$

Costs in the Short Run

Fixed cost:

- **any cost that does not depend on the firm's level of output.** (The firm incurs these costs even if it doesn't produce any output).
- **firms have no control over fixed costs in the short run.** (For this reason, fixed costs are sometimes called sunk costs).
 - **obvious examples:** property taxes, loan payments, etc.
 - **not-so-obvious example:** firm must pay "rent" to hired capital. If that level of capital cannot be adjusted immediately ("fixed factor"), then rental payments are a fixed cost in the short-run

Variable cost:

- **depends on the level of production**
- **derived from production requirements and input prices**
 - **variable cost rises as output rises because firm has to hire more inputs** (capital and labor) **to produce larger quantities of output**
 - (in the Long Run all costs are variable)

Marginal Cost

Marginal cost:

- **increase in total cost from producing one more unit of output** (the additional cost of inputs required to produce each successive unit of output)
- **only reflects changes in variable costs**
 - fixed cost does not increase as output increases
 - marginal cost is the slope of both total cost and variable cost

Shape of the Marginal Cost Curve

In the short run, the firm is constrained by a fixed input, therefore:

1. the firm faces diminishing returns to variable inputs and
2. the firm has limited capacity to produce output

As the firm approaches that capacity it becomes increasingly costly to produce successively higher levels of output. Marginal costs ultimately increase with output in the short run.

Short-Run Average and Marginal Cost

- If a firm's capital stock is fixed in the short-run, then the rental payments that the firm makes on its capital stock is a fixed cost.
- We can use that assumption to derive short-run average and marginal cost curves.
- So start by assuming that a firm's production function is given by:

$$X = K^{2/3} L^{1/3}$$

- Since the firm's capital stock is fixed (by assumption) we can solve the production function for labor to find the amount of labor needed to produce various levels of output:

$$L = \frac{X^3}{K^2}$$

- Its total costs are given by:

$$TC = rK + wL$$

$$TC = rK + w \frac{X^3}{K^2}$$

Short-Run Average and Marginal Cost

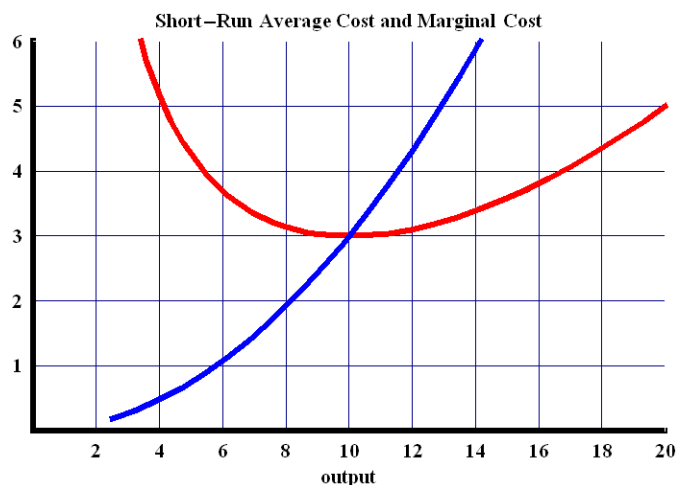
To find Short-Run Average Cost simply divide by Total Cost by X:

$$AC = \frac{TC}{X} \Rightarrow AC = \frac{rK}{X} + w \frac{X^2}{K^2}$$

To find Short-Run Marginal Cost take the derivative of Total Cost with respect to X:

$$MC \equiv \frac{dTC}{dX} \Rightarrow MC = 3w \frac{X^2}{K^2}$$

So if the wage rate is \$1 per unit of labor, i.e. $w = \$1$, and the rental rate is \$2 per unit of capital, i.e. $r = \$2$, and the firm has a capital stock of 10 units, then:

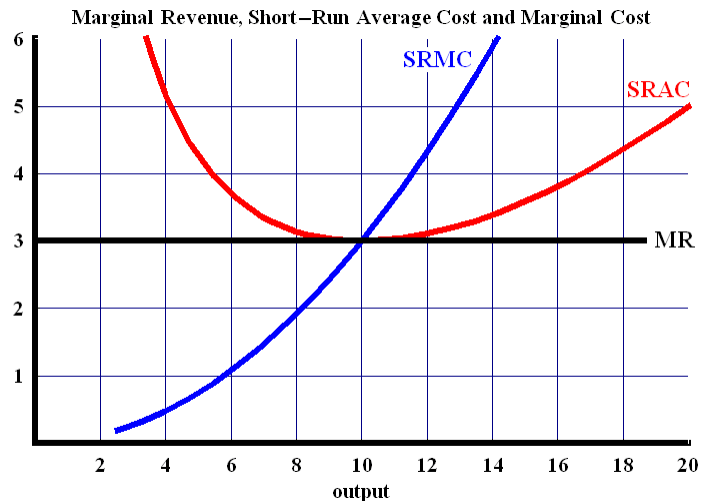


Revenue, Costs and Profit-Maximization

- **Total Revenue** – amount that firm receives from sale of its output
- **Marginal Revenue** – additional revenue that a firm takes in when it increases output by one additional unit.
- For example, if a firm is operating in a competitive industry – so that it has no ability to influence the market price – and if the market price is \$3, then:

Comp. Firm's $MR = p^*$

p	Q	TR	MR
3	0	0	3
3	1	3	3
3	2	6	3
3	3	9	3
3	4	12	3
3	5	15	3
3	6	18	3



- Profit-maximizing level of output occurs where a firm's $MR = MC$
- Since $MR = p^*$, it will produce up to the point where $p^* = MC$

But wait ...

- In the example above, the firm is making zero (economic) profit
- If it produces up to the point where $p^* = MC$, then:
 - it will produce 10 units of output and
 - sell it at a price of \$3, for total revenue of \$30
 - but it's total costs will also be \$30,
 - since at an output of 10 units, its average cost (per unit) is \$3
 - and **average cost times output equals total cost**
 - so its total revenue equals its total cost

Could a firm ever make positive (economic) profit? Yes.

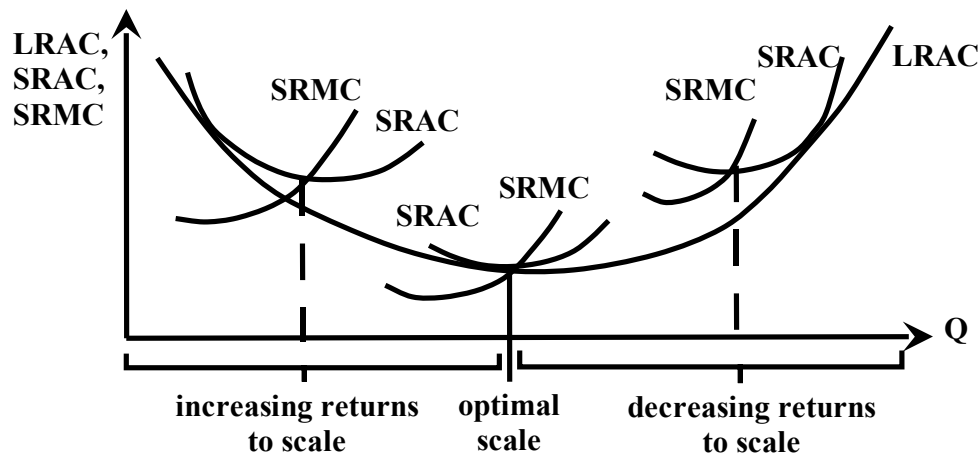
- If the price were higher (say: \$4), then it would make positive (economic) in the **SHORT RUN**, but
- in the **LONG RUN** new firms will enter the market and push down the market price until the firm's profit is zero
- It could also make positive (economic) profit if it's a monopoly

Long-Run Costs: Returns to Scale

- In the **SHORT RUN**, firms have to decide how much to produce in the current scale of plant (factory size is fixed).
- In the **LONG RUN** firms, have to choose among many potential scales of plant (they can expand the factory).
- **Increasing returns to scale** (or economies of scale), refers to an increase in a firm's scale of production, which leads to lower average costs per unit produced.
- **Constant returns to scale** refers to an increase in a firm's scale of production, which has no effect on average costs per unit produced.
- **Decreasing returns to scale** (or diseconomies of scale) refers to an increase in a firm's scale of production, which leads to higher average costs per unit produced.

Long-Run Average Cost Curve

- The Long-Run Average Cost (LRAC) curve shows the different scales on which a firm can operate in the long-run. Each scale of operation defines a different short-run.
- The Long-Run Average Cost curve of a firm:
 - is downward-sloping when the firm exhibits increasing returns to scale.
 - is upward sloping when the firm exhibits decreasing returns to scale.
- The optimal scale of plant is the scale that minimizes long-run average cost.

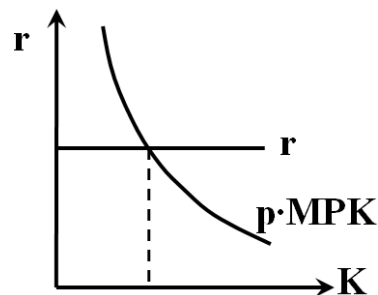


Returns to Scale

- **As mentioned previously, the only two cases where a firm can make positive profit is:**
 - if the firm is a monopoly
 - if there is not enough competition to drive the price down to the point where competitive firms make zero profit
- **Monopoly arises when the firm faces increasing returns to scale over the whole range of output**
 - So if the monopolist were to double his/her inputs of capital and labor, then his/her output would more than double
 - So his/her costs would double, but he/she would be able to sell more than two times the amount of output
- **The production process of a competitive firm faces exhibits constant returns to scale**
 - So if the firm were to double its inputs of capital and labor, then its output would exactly than double
 - So its costs would double and it would be able to sell exactly two times the amount of output

Factor Demand in the Long Run

- **When we derived the firm's short-run marginal and average cost functions, we assumed that the firm was operating under a fixed scale of production.**
- **Specifically, we assumed that the firm's capital stock was held fixed.**
- **So when we derived the condition for profit-maximization, we focused on the condition that:**
 - the firm hires labor up to the point where the wage equals the price times the marginal product of labor (MPL): $w = p \cdot MPL$
- **In the long run, the other condition must also hold:**
 - firm hires capital up to the point where the rental rate on capital equals the price times the marginal product of capital (MPK): $r = p \cdot MPK$



Zero Profit in the Long Run

Now let's bring it all together.

- Firm's profit is given by: $\Pi = p \cdot K^{2/3} \cdot L^{1/3} - r \cdot K - w \cdot L$
- Firm's factor demands are given by: $r = p \cdot MPK$ and $w = p \cdot MPL$
- Using the calculus tricks that you learned:
 - $MPK = \frac{2}{3} \cdot K^{-1/3} \cdot L^{1/3}$ and $MPL = \frac{1}{3} \cdot K^{2/3} \cdot L^{-2/3}$
- Plugging the factor demands into the profit function:

$$\begin{aligned}\Pi &= p \cdot K^{2/3} \cdot L^{1/3} - \left(p \cdot \frac{2}{3} \cdot K^{-1/3} \cdot L^{1/3} \right) \cdot K - \left(p \cdot \frac{1}{3} \cdot K^{2/3} \cdot L^{-2/3} \right) \cdot L \\ &= p \cdot \left(K^{2/3} \cdot L^{1/3} \right) - \frac{2}{3} \cdot p \cdot \left(K^{2/3} \cdot L^{1/3} \right) - \frac{1}{3} \cdot p \cdot \left(K^{2/3} \cdot L^{1/3} \right) = 0\end{aligned}$$

Zero Profit in the Long Run

So what does this mean?

- If the firm operates in a competitive industry
 - with a large number of firms
 - so that it cannot affect market prices
- if the firm's production process exhibits **constant returns to scale**, and
- if the firm varies capital and labor optimally, in order to maximize (economic) profit, then
- its maximum (economic) profit will be zero in the long run
- even though it is producing at the lowest possible cost per unit of output (lowest possible average cost)