Notes on Profit Maximization

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These notes are a supplement to a lecture that I will deliver in class. They contain the tables that I will use to provide a simple, numerical example of profit-maximization. They do not contain much explanation, so please do not treat them as a substitute for the lecture.

1 A Firm's Profit

A firm's profit, Π, is equal to the difference between its total revenue and its total costs:

$$\Pi = TR - TC \tag{1}$$

For the purposes of this lecture, we will assume that the firm is in perfect competition and, therefore, cannot affect the market equilibrium price of the good that it produces. It simply takes the price as given. We will also assume that it can sell any quantity that it desires at the market price. These assumptions imply that total revenue is equal to price times quantity sold:

$$TR = p \cdot Q \tag{2}$$

We will also make the simplifying assumption that the only input into the production of the good that it sells is labor, so that its total costs are equal to the wage rate times the amount of labor that it employs:

$$TC = w \cdot L \tag{3}$$

2 Production Assumptions

Because labor is the only input into the production process, the quantity that the firm produces depends only on the amount of labor that it employs. We will assume that the firm faces **diminishing marginal returns** to the labor that it employs. In other words, "as you add more and more cooks to the kitchen, the additional amount of food produced by each additional cook falls."

To formalize this concept, we will assume that the quantity that the firm produces is proportional to the square root of the amount of labor that it employs:

$$Q = A \cdot \sqrt{L} \tag{4}$$

Its **marginal product of labor** is the additional quantity that it produces when it employs an additional unit of labor:

$$MPL \equiv \frac{\Delta Q}{\Delta L} \tag{5}$$

Those of you who have taken a course in calculus may notice that the marginal product of labor is the derivative of quantity produced with respect to labor:

$$\frac{dQ}{dL} = \frac{A}{2} \cdot \frac{1}{\sqrt{L}} \tag{6}$$

Finally, if the firm wished to produce a particular quantity of output, that choice would determine how much labor it employs:

$$L = \left(\frac{Q}{A}\right)^2 \tag{7}$$

3 Profit Maximization

Now that we have defined a firm's profit and production assumptions, we will use a simple, numerical example to find the quantity of output that maximizes its profit and the amount of labor it would employ to maximize its profit.

Specifically, we'll assume that the market price of the firm's output is $4 per unit, the wage rate is $10 per unit of labor and the scale factor in its production is 10. In other words:

- $p = 4$
- $w = 10$
- $A = 10$

3.1 Brute-Force Method

One way to find the profit-maximizing levels of output and labor is to compute the firm's profit at each level of output:

Table 1: Brute-Force Method

Q	L	TR	TC	Π
10	1	40	10	30
20	**4**	80	40	**40**
30	9	120	90	30
40	16	160	160	0

This method shows that the firm would maximize profit by employing 4 units of labor to produce 20 units of output.

3.2 Cost-Benefit Analysis

Another way to find the profit-maximizing levels of output and labor is to compare the benefits associated with producing an additional unit of output (in terms of increased revenue) with the cost of producing an additional unit of output.

If producing an additional unit of output would bring in additional revenue that exceeds the additional cost associated with producing another unit, then the firm would increase its profit by producing that additional unit. By contrast, if the additional revenue were less than the additional cost, then the firm's profit would fall if it produced that additional unit.

A firm's **marginal revenue** is the additional revenue that it receives when it sells an additional unit of output:

$$MR \equiv \frac{\Delta TR}{\Delta Q} \tag{8}$$

Its **marginal cost** is the additional cost that it incurs by producing an additional unit of output:

$$MC \equiv \frac{\Delta TC}{\Delta Q} \tag{9}$$

The firm maximizes profit by producing output up to the point at which marginal revenue equals marginal cost.

In Table 1, we saw that the firm maximizes profit by producing 20 units of output. Table 2 shows that the firm's marginal revenue exceeds marginal cost when it produces fewer than 20 units. Table 2 also shows that the firm's marginal revenue is less than marginal cost when it produces more than 20 units.

Table 2: MR = MC Method

Q	L	TR	MR	MC	TC
10	1	40			10
			$40/10 = 4$	$3 = 30/10$	
20	4	80			40
			$40/10 = 4$	$5 = 50/10$	
30	9	120			90
			$40/10 = 4$	$7 = 70/10$	
40	16	160			160

In other words, it would not make sense for the firm to produce 15 units. Producing an additional unit would bring in additional revenue that exceeds the additional cost associated with producing it, so it should produce more than 15 units.

Similarly, it would not make sense for the firm to produce 25 units. Producing an additional unit would bring in additional revenue that falls short of the additional cost associated with producing it. Notice also that if it reduced production by one unit, the subtracted revenue would be less than the subtracted cost, so it should produce fewer than 25 units. (Ideally, it should produce 20 units).

3.3 Cost-Benefit Analysis, One More Time

Another form of cost-benefit analysis compares the cost of hiring an additional unit of labor to the additional benefit (in term of increased revenue) that hiring an additional unit of labor brings to the firm.

In this case, the additional cost of employing an additional unit of labor (i.e. the marginal cost) is simply the wage rate, w. The additional revenue is the price of the firm's output times the additional output that is produced when an additional unit of labor is hired:

$$p \cdot \frac{\Delta Q}{\Delta L} \equiv p \cdot MPL \tag{10}$$

The term $p \cdot MPL$ is usually called the **"marginal revenue product of labor,"** but some economists call it the "marginal value product of labor."

If employing an additional unit of labor would bring in additional revenue that exceeds the wage rate (i.e. the additional cost associated with another unit of labor), then the firm would increase its profit by hiring that additional unit of labor. By contrast, if the additional revenue were less than the wage rate, then the firm's profit would fall if it hired that additional unit.

To maximize profit, the firm will hire labor up to the point at which the marginal revenue product of labor is equal to the wage rate:

$$w = p \cdot MPL \tag{11}$$

Table 3: w = p MPL Method

Q	L	TR	$p \cdot MPL$	w
10	1	40		
			$40/3 \approx 13.3$	10
20	4	80		
			$40/5 = 8.00$	10
30	9	120		
			$40/7 \approx 5.71$	10
40	16	160		

In Table 1, we saw that the firm maximizes profit by employing 4 units of labor. Table 3 shows that the marginal revenue product of labor is greater than the wage rate when it employs fewer than 4 units of labor. Table 3 also shows that the marginal revenue product of labor is less than the wage rate when it employs more than 4 units.

In other words, it would not make sense for the firm to employ 2 units of labor. Hiring an additional unit would bring in additional revenue that exceeds the wage rate (i.e. the additional cost associated with hiring another unit of labor), so it should employ more than 2 units.

Similarly, it would not make sense for the firm to employ 6 units of labor. Hiring an additional unit would bring in additional revenue that falls short of the wage rate. Notice also that if it reduced the amount of labor that it employs by one unit, the subtracted cost would be less than the subtracted revenue, so it should employ fewer than 6 units. (Ideally, it should employ 4 units).