MACRO GRAD

5. Feb. 2009

Intro to Solow Model

$$Y = F(K, AL) \qquad \begin{array}{l} K = K(t) \\ A = A(t) \\ L = L(t) \end{array}$$

AL is "effective labor"
technology augments labor — Harrod-Neutral

production fn exhibits constant returns to scale
→ if double all inputs, output doubles

$$c \cdot F(K, AL) = F(cK, cAL) \quad \forall\ c \geq 0$$

if $c = \frac{1}{AL}$ then

$$F\left(\frac{K}{AL}, 1\right) = \frac{1}{AL} F(K, AL)$$

so define $k \equiv K/AL$ and $y \equiv Y/AL$

$$f(k) \equiv F(k, 1)$$

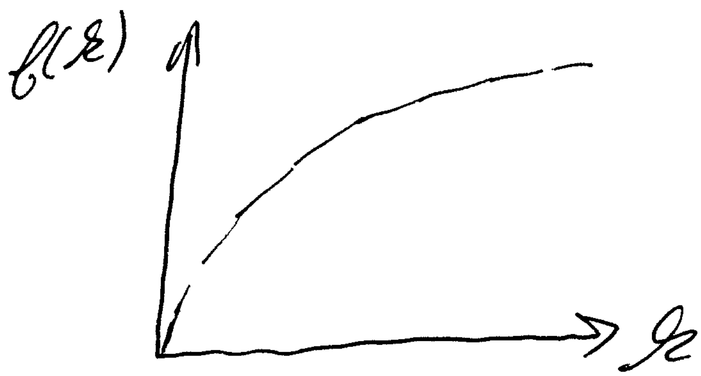
$$y = f(k)$$

$$y = f(k)$$

output per eff labor is a fn of capital per eff labor

$f(k)$ exhibits diminishing marginal returns

$f(0) = 0 \qquad f'(k) > 0 \qquad f''(k) < 0$



we'll use

$$Y = K^{\alpha} (AL)^{1-\alpha}$$

$$y = k^{\alpha}$$

recall that

$$K = K(t) \qquad A = A(t) \qquad L = L(t)$$

fns of time

growth rates

$$g \equiv \frac{\dot{A}}{A} \qquad n \equiv \frac{\dot{L}}{L}$$

$$A(t) = A(0)e^{gt} \qquad L(t) = L(0)e^{nt}$$

$$\dot{A} \equiv \frac{d\,A(t)}{dt} = g\,A(0)e^{gt} \qquad \dot{L} = n\,L(0)e^{nt}$$

saving rate, rate of depreciation

$$\dot{K} = sY - \delta K$$

Calculus Tricks

the percentage change in a ~~ratio~~ product is ~~eq~~ equal to the ~~difference~~ sum of the percentage changes

$$\frac{(\dot{AL})}{AL} = \frac{\dot{A}}{A} + \frac{\dot{L}}{L} \qquad \Big/\!\!\Big/ \qquad \frac{(\dot{K/AL})}{K/AL} = \frac{\dot{K}}{K} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L}$$

the percentage change in a ratio is equal to the difference between the percentage changes

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L}$$

$$= \frac{sY}{K} - \frac{\delta K}{K} - g - n$$

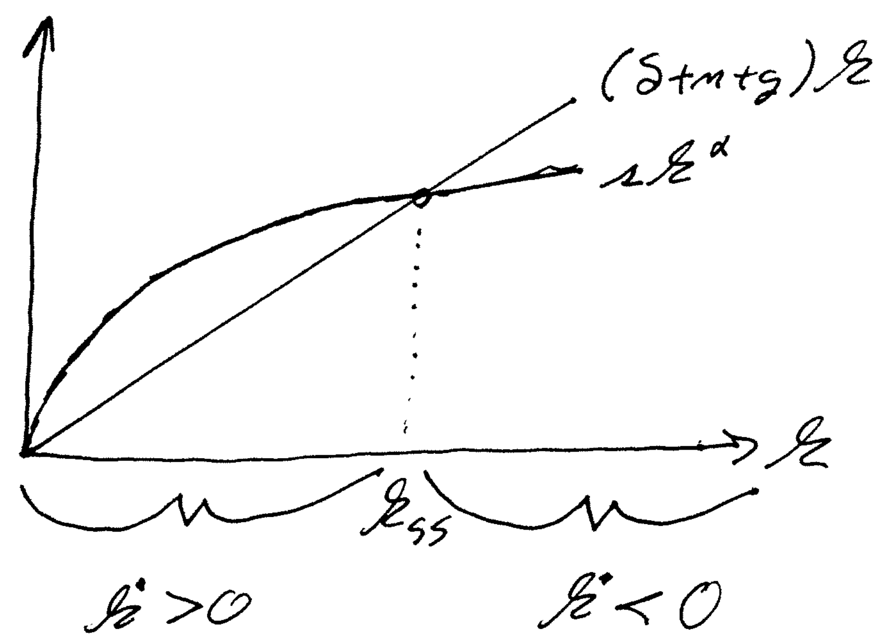
$$\frac{Y}{K} = \frac{Y/AL}{K/AL} = \frac{y}{k} = \frac{k^{\alpha}}{k} = k^{\alpha-1}$$

$$\frac{\dot{k}}{k} = s k^{\alpha-1} - (\delta + n + g)$$

$$\dot{k} = s k^{\alpha} - (\delta + n + g) k$$

the change in the stock of capital per effective labor is equal to the difference betw/n saving per eff labor and rate at which capital must be replaced to keep capital per eff labor constant times capital per eff labor

Key Eqm $\dot{k} = s k^\alpha - (\delta + n + g) k$



$k_{ss} \equiv$ steady state stock of capital per effective labor

when $\dot{k} = 0$, then $s k^\alpha = (\delta + n + g) k$

in SS constant capital-output ratio

$$\frac{K}{Y} = k_{ss}^{1-\alpha} = \frac{s}{\delta + n + g}$$

$$k_{ss} = \left(\frac{s}{\delta + n + g}\right)^{\frac{1}{1-\alpha}} \qquad y_{ss} = \left(\frac{s}{\delta + n + g}\right)^{\frac{\alpha}{1-\alpha}}$$

$$k_{ss} = \left(\frac{s}{\delta+n+g}\right)^{\frac{1}{1-\alpha}} \qquad y_{ss} = \left(\frac{s}{\delta+n+g}\right)^{\frac{\alpha}{1-\alpha}}$$

notice however that in SS

output per worker $\frac{Y}{L}$

does NOT stop growing

output per unit of eff labor stops growing, but ~~we don't consume~~ our living standards depend on output per worker

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L}$$

in SS $\dot{k} = 0$

$$\frac{\dot{(K/L)}}{K/L} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} = \frac{\dot{A}}{A} = g$$

$\dot{y} = 0$ (in SS) because $\dot{k} = 0$

$$\frac{\dot{y}}{y} = \frac{\dot{Y}}{Y} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L}$$

$$\frac{\dot{(Y/L)}}{Y/L} = \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = \frac{\dot{A}}{A} = g$$