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New Growth Theory

<u>Solow</u>

$$\left.\begin{aligned} y &\equiv \frac{Y}{L} \\ \tilde{y} &\equiv \frac{Y}{AL} \end{aligned}\right\} \quad \frac{\dot{\tilde{y}}}{\tilde{y}} = \frac{\dot{Y}}{Y} - g - n \quad \left\{\begin{aligned} g &\equiv \frac{\dot{A}}{A} \\ n &\equiv \frac{\dot{L}}{L} \end{aligned}\right.$$

$$\boxed{\frac{\dot{y}}{y} = \frac{\dot{\tilde{y}}}{\tilde{y}} + g}$$

In SS, $\dot{\tilde{y}} = 0$, so growth rate of output per worker is equal to rate of technological progress

But what determines the rate of technological progress?

→ is it exogenous? (as Solow + MRW assume)

→ does it depend on the share of individuals in the R+D sector?

→ does it depend on capital accumulation?

Model in which g a fn of R+D

goods producing sector produces all output and fractions $(1-a_K)$ and $(1-a_L)$ of capital stock and labor force respectively allocated to goods production

fractions a_K and a_L allocated to R+D

$$Y = \left((1-a_K)K\right)^{\alpha}\left(A(1-a_L)L\right)^{1-\alpha}$$

$$\frac{\dot{A}}{A} = B\,(a_K K)^{\beta}(a_L L)^{\gamma} A^{\theta-1}$$

$\beta+\gamma$ not necessarily equal to one, so there may be increasing returns to scale in R+D or decreasing returns to scale in R+D

Assume: $n \equiv \frac{\dot{L}}{L}$ and $\delta = 0$

→ Simple case in which capital is not a productive factor

$$\alpha = 0 \quad \& \quad \beta = 0$$

→ implies that

$$Y = A(1 - a_L)L$$

$$g_A \equiv \frac{\dot{A}}{A} = B\, a_L^{\gamma}\, L^{\gamma} A^{\theta - 1}$$

→ at what rate does the rate of technological progress grow?
(i.e. "what's the growth rate of the growth rate?")

$$\ln g_A = \ln B + \gamma \ln a_L + \gamma \ln L + (\theta - 1) \ln A$$

$$\boxed{\frac{\dot{g}_A}{g_A} \equiv \frac{d \ln g_A}{dt} = \gamma n + (\theta - 1) g_A}$$

if $\theta < 1$, then there's a steady state

$$\boxed{g_{A,ss} = \frac{\gamma n}{1 - \theta}}$$

if $\theta < 1$ $\quad g_{A,ss} = \dfrac{\gamma n}{1-\theta}$

that's an interesting result

The growth rate is an increasing function of the population growth rate. Kinda weird, but makes a bit of sense if we think of it as a model of worldwide economic growth (more people to make discoveries that can be used anywhere around the world)

if $\theta > 1$ growth rate an increasing function of the growth rate (ridiculous!)

if $\theta = 1$ growth rate of growth rate constant (if $n > 0$) (still ridiculous!)

Ok, so what if

$\alpha > 0$ & $\beta > 0$?

Recall: $\frac{\dot{A}}{A} = B\,(a_K K)^\beta (a_L L)^\gamma A^{\theta-1}$

$\delta = 0 \Rightarrow \boxed{\dot{K} = sY}$ // $n \equiv \frac{\dot{L}}{L}$

$$\dot{K} = s\left((1-a_K)K\right)^\alpha \left(A(1-a_L)L\right)^{1-\alpha}$$

$$\boxed{g_K \equiv \frac{\dot{K}}{K} = s(1-a_K)^\alpha (1-a_L)^{1-\alpha}\left(\frac{K}{AL}\right)^{\alpha-1}}$$

$$\boxed{g_A \equiv \frac{\dot{A}}{A} = B(a_K K)^\beta (a_L L)^\gamma A^{\theta-1}}$$

$\alpha < 1$

so g_K a decreasing fn of capital per eff. labor

Once again we need to find the growth rate of the growth rate

$$\frac{\dot{g}_K}{g_K} = (\alpha-1)(g_K - g_A - n)$$

$$= (1-\alpha)(n + g_A - g_K)$$

$(+)$

in SS $\dot{g}_K = 0 \Rightarrow g_{K,SS} = n + g_A$

$$\frac{\dot{g}_A}{g_A} = \beta g_K + \gamma n + (\theta - 1) g_A$$

Notice that growth rates only converge to a steady state if $\beta + \theta < 1$

when $\dot{g}_K = 0$

$$\frac{\dot{g}_A}{g_A} = (\beta+\gamma)n + (\beta+\theta-1) g_A$$

if $\beta + \theta < 1$

then $\dot{g}_A = 0$ and $\dot{g}_K = 0$

where
$$\boxed{\begin{aligned} g_{A,SS} &= \frac{(\beta+\gamma)n}{1-(\beta+\theta)} \\ g_{K,SS} &= n\left(\frac{1-\theta+\gamma}{1-\beta-\theta}\right) \end{aligned}}$$

notice that both growth rates are an increasing function of the population growth rate

cases where $\beta + \theta \geq 1$

imply explosive growth

(ridiculous)

How to devote labor & capital to R&D?

knowledge is "nonrival"

→ If I can use the knowledge so can you

Consumer goods by contrast are "rival" – we can't both eat one whole apple

→ In such a case rental price of knowledge is zero

→ creation of knowledge cannot be motivated by private gain

some knowledge is "excludable" however

→ patent laws – gov't grant of monopoly on use of design or discovery (horrible!)

→ copyright law – gov't grant of monopoly on a verbatim copy (not so bad)

To create incentives for knowledge creation:

→ gov't support for its production

→ patents – Patent holder has exclusive control over use of an idea. He/she may license that use to someone else (or may decide not to). Invitation for abuse.

→ alternative opportunities for talent
Talented individuals may engage in knowledge creation OR rent-seeking

"$Y = AK$" model ← ~~"Learning by doing"~~

→ As we engage in an activity we may look for ways to do it more efficiently, so knowledge develops as we produce new capital

$$A = BK^{\phi} \qquad B \text{ constant}$$

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$$Y = K^{\alpha}(AL)^{1-\alpha}$$

$$A = BK^{\phi}$$ B constant

$$\boxed{Y = B^{1-\alpha} K^{\phi+\alpha(1-\phi)} L^{1-\alpha}}$$

if $\delta = 0 \Rightarrow \dot{K} = sY$

$$\boxed{\frac{\dot{K}}{K} = s\left(\frac{K^{1-\phi}}{BL}\right)^{\alpha-1}}$$

$$\boxed{\frac{\dot{A}}{A} = \phi\frac{\dot{K}}{K}}$$

so technological progress → increasing fn of the saving rate

$$g_K \equiv \frac{\dot{K}}{K} \qquad \frac{\dot{g}_K}{g_K} = (\alpha-1)(1-\phi)\frac{\dot{K}}{K} + (1-\alpha)n$$

→ rate of growth of growth rate an increasing function of rate of labor force growth

→ If $n>0$ and $0<\phi<1$ then convergence to constant growth rate

$$\frac{\dot{g}_K}{g_K}=0 \Rightarrow g_{K,ss}=\frac{n}{1-\phi}$$

→ But if $\phi>1$ growth rate does not converge + we have explosive growth (ridiculous)

→ More interesting is the case where $\phi=1$ and $n=0$

$$\text{then } \frac{\dot{A}}{A}=\frac{\dot{K}}{K}=s\,\underbrace{(BL)^{1-\alpha}}_{\text{constant}}$$

note that production function in such a model would be:

$$Y=K\underbrace{(BL)^{1-\alpha}}_{\text{constant}}$$

So growth rate of output per worker would be proportionate to saving rate

Romer combines

$$Y = K(BL)^{1-\alpha}$$

with Ramsey Model to show that

$\boxed{\text{private MPK} < \text{social MPK}}$

implies that socially optimal growth rate greater than the one that occurs in decentralised equilibrium (justification for subsidising capital accumulation if model a good description of economy)

Private firm's production function:

$$Y_i = B^{1-\alpha} K^{1-\alpha} K_i^{\alpha} L_i^{1-\alpha}$$

in equilibrium K/L equated across firms, so

private $MPK_i = \alpha (BL)^{1-\alpha} \quad \forall\ i$

which is less than the

social $MPK = (BL)^{1-\alpha}$