GRADUATE MICRO
BARUCH COLLEGE

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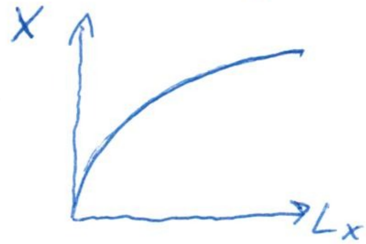
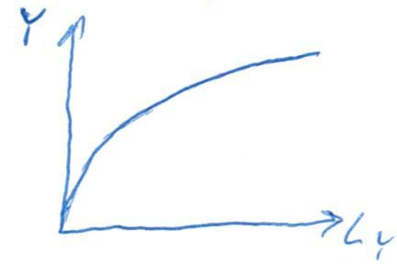
Notes on Part One of Problem Set

Robinson's PPF

$X = \sqrt{L_X}$ $\qquad$ $Y = \sqrt{L_Y}$





Production Functions exhibit DIMINISHING MARGINAL RETURNS

therefore

PPF will exhibit INCREASING OPPORTUNITY COST

$$\bar{L} = L_X + L_Y \quad \Leftarrow \text{labor constraint}$$

$$\bar{L} = X^2 + Y^2$$

$$Y = \sqrt{\bar{L} - X^2} \quad \Leftarrow \text{PPF}$$

when Labor held constant at $\bar{L} = 128$, then:

$$Y = \sqrt{128 - X^2}$$

Robinson's UTILITY

$$U = \frac{1}{\frac{1}{X} + \frac{1}{Y}}$$

indifference curve holds utility constant at $U^* = 4$

$\Leftarrow$ MAXIMUM UTILITY Robinson can attain

$$Y = \frac{1}{\frac{1}{U} - \frac{1}{X}}$$

$\sigma_{XY} = \frac{1}{2}$

X and Y are highly COMPLEMENTARY

$$Y = \frac{1}{\frac{1}{4} - \frac{1}{X}}$$

INDIFFERENCE CURVE at UTILITY MAX

Robinson's PPF

$$Y = \sqrt{128 - X^2}$$

Robinson's Relative Supply Schedule

X	Y	OPP COST of X $-1 * \frac{\Delta Y}{\Delta X}$	REL QTY of X $\frac{X}{Y}$	
6,0	9,59		0,63	
6,5		0,70		
7,0	8,89		0,79	
7,5	8,47	0,89		
8,0	8,00	1,00	1,00	equilibrium
8,5	7,47	1,14		
9,0	6,86		1,31	
9,5		1,55		
10,0	5,29		1,89	

UTILITY MAX

Robinson's Indifference Curve

$$Y = \frac{1}{\frac{1}{4} - \frac{1}{X}}$$

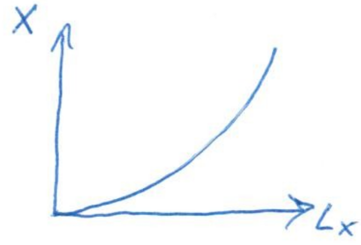
Robinson's Relative Demand Schedule

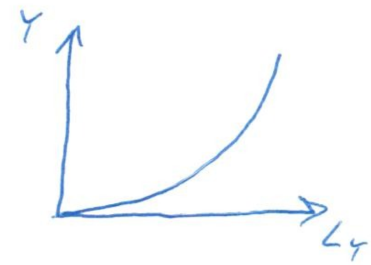
X	Y	MRS $-1 * \frac{\Delta Y}{\Delta X}$	REL QTY of X X/Y	
6,0	12,00		0,50	
6,5		2,67		
7,0	9,33		0,75	
7,5	8,57	1,33		
8,0	8,00	1,00	1,00	equilibrium
8,5	7,56	0,80		
9,0	7,20		1,25	
9,5		0,53		
10,0	6,67		1,50	

UTILITY MAX

FRIDAY'S PPF

$X = L_x^2$ $\qquad$ $Y = L_Y^2$





Production Functions exhibit INCREASING MARGINAL RETURNS

therefore

PPF will exhibit DECREASING OPPORTUNITY COST

$\bar{L} = L_X + L_Y$ ← labor constraint

$\bar{L} = \sqrt{X} + \sqrt{Y}$

$Y = (\bar{L} - \sqrt{X})^2$ ← PPF

when Labor held constant at $\bar{L} = 4$, then: $Y = (4 - \sqrt{X})^2$

FRIDAY'S UTILITY

$U = (\sqrt{X} + 2 \cdot \sqrt{Y})^2$

$\sigma_{XY} = 2$
X and Y are HIGHLY SUBSTITUTABLE

to derive indifference curve, we'll hold utility constant at $\boxed{U^* = 64}$

2 MAXIMUM UTILITY Friday can attain

$U = (\sqrt{X} + 2\sqrt{Y})^2$

$\sqrt{U} = \sqrt{X} + 2\sqrt{Y}$

$\sqrt{Y} = \frac{1}{2}(\sqrt{U} - \sqrt{X})$

$Y = \dfrac{(\sqrt{U} - \sqrt{X})^2}{4}$ ← indiff curve

indiff curve when $U^* = 64$	$Y = \dfrac{(8 - \sqrt{X})^2}{4}$

FRIDAY'S RELATIVE SUPPLY SCHEDULE

p. 4

FRIDAY'S PPF

$$Y = (4 - \sqrt{X})^2$$

X	Y	OPP COST of X $-1 * \frac{\Delta Y}{\Delta X}$	REL QTY of X X/Y	
0,0	16,0	+ ∞	0,0	equilibrium
0,5		5,00		
1,0	9,0		0,11	
1,5		2,31		
2,0	6,69		0,30	
2,5		1,55		
3,0	5,14		0,58	
3,5		1,14		
4,0	4,00		1,00	

UTILITY MAX

FRIDAY'S RELATIVE DEMAND SCHEDULE

FRIDAY'S INDIFF CURVE

$$Y = \frac{1}{4}(8 - \sqrt{X})^2$$

X	Y	MRS $-1 * \frac{\Delta Y}{\Delta X}$	REL QTY of X X/Y	
0,0	16,0	+ ∞	0,0	equilibrium
0,5		3,75		
1,0	12,25		0,08	
1,5		1,41		
2,0	10,84		0,18	
2,5		1,02		
3,0	9,82		0,31	
3,5		0,82		
4,0	9,00		0,44	

UTILITY MAX

Questions 3 + 6

→ Robinson maximizes his utility (subject to the "production constraint") by producing + consuming

$$X = 8 \quad \text{and} \quad Y = 8$$

→ Friday maximizes his utility by producing + consuming

$$X = 0 \quad \text{and} \quad Y = 16$$

→ Robinson's PPF is "concave" - it exhibits increasing opportunity cost because Robinson's production functions exhibit diminishing marginal returns

→ Friday's PPF is "convex" - it exhibits decreasing opportunity cost because Friday's production functions exhibit increasing marginal returns

→ BECAUSE Robinson's PPF exhibits increasing opp cost Robinson's Relative Supply curve slopes upward

→ Similarly Friday's PPF exhibits decreasing opp cost therefore Friday's Rel Supply curve slopes downward

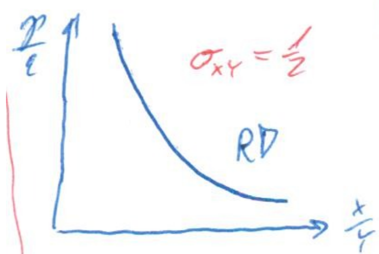
→ FRIDAY's indifference curve is "flatter" than Robinson's

- FRIDAY is more willing to substitute X for Y than ROBINSON

FRIDAY's Rel Demand

- in this model, Friday's elasticity of substitution in consumption is constant and equal to $\sigma_{XY} = 2$

Robinson's Rel Demand

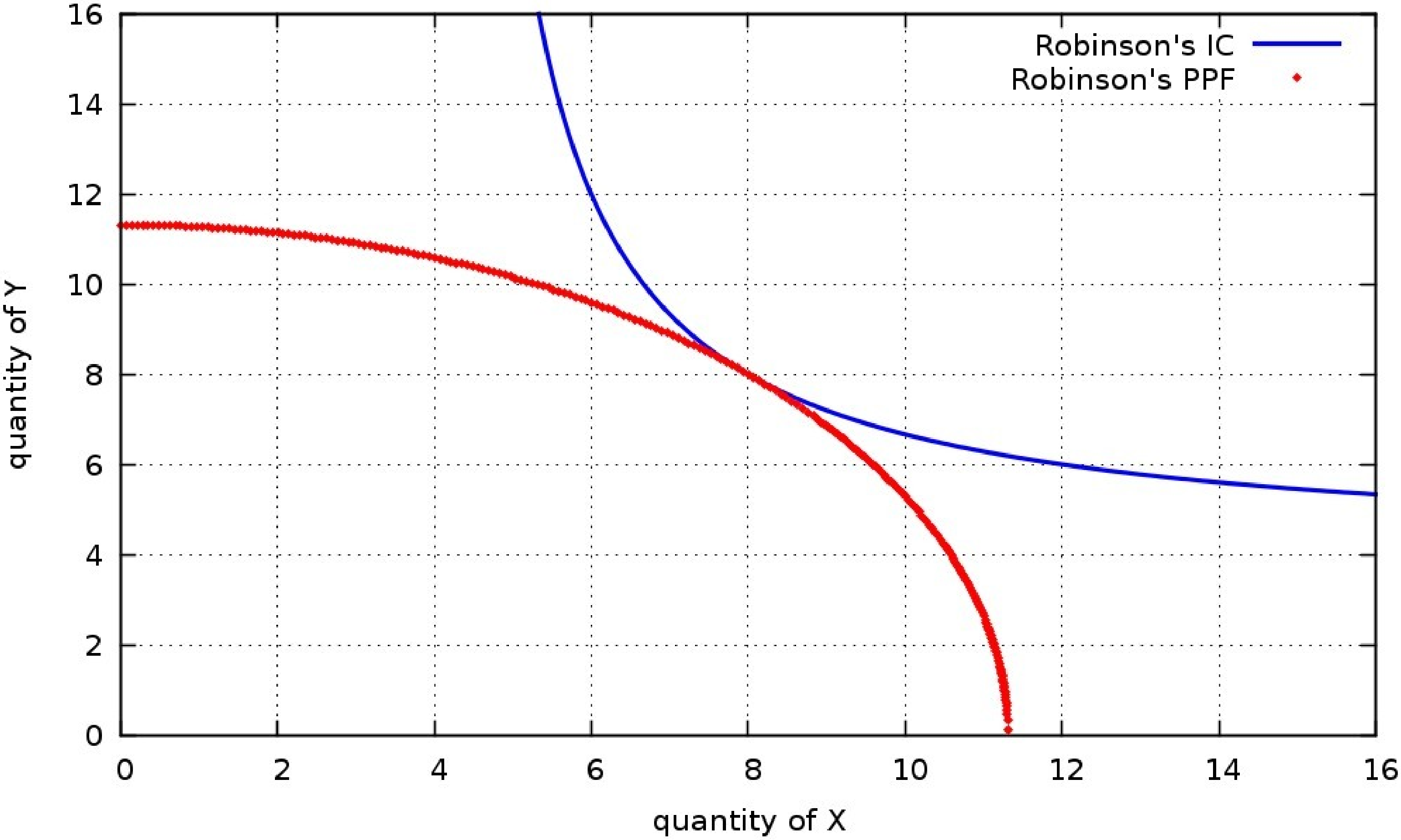


- Robinson's elasticity of substitution in consumption is also constant, but only equal to $\sigma_{XY} = \frac{1}{2}$

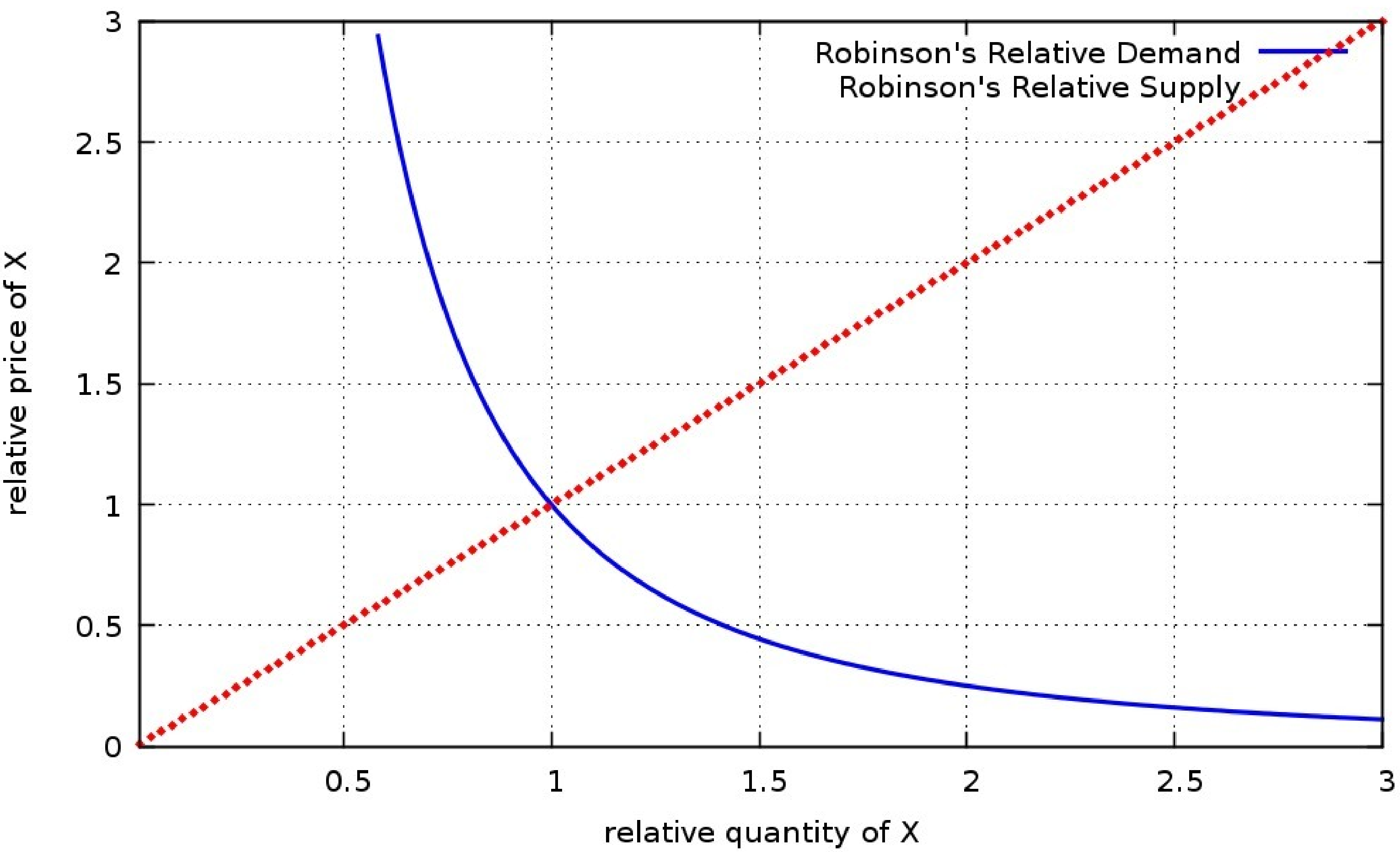
- Friday's relative demand curve is ("flatter") more elastic than Robinson's because Friday is more willing to substitute x for y than Robinson

→ In equilibrium, Robinson produces + consumes one X for each Y (equilibrium relative quantity of X) and he is willing to exchange X and Y at a rate of one Y per X (equilibrium relative price of X)

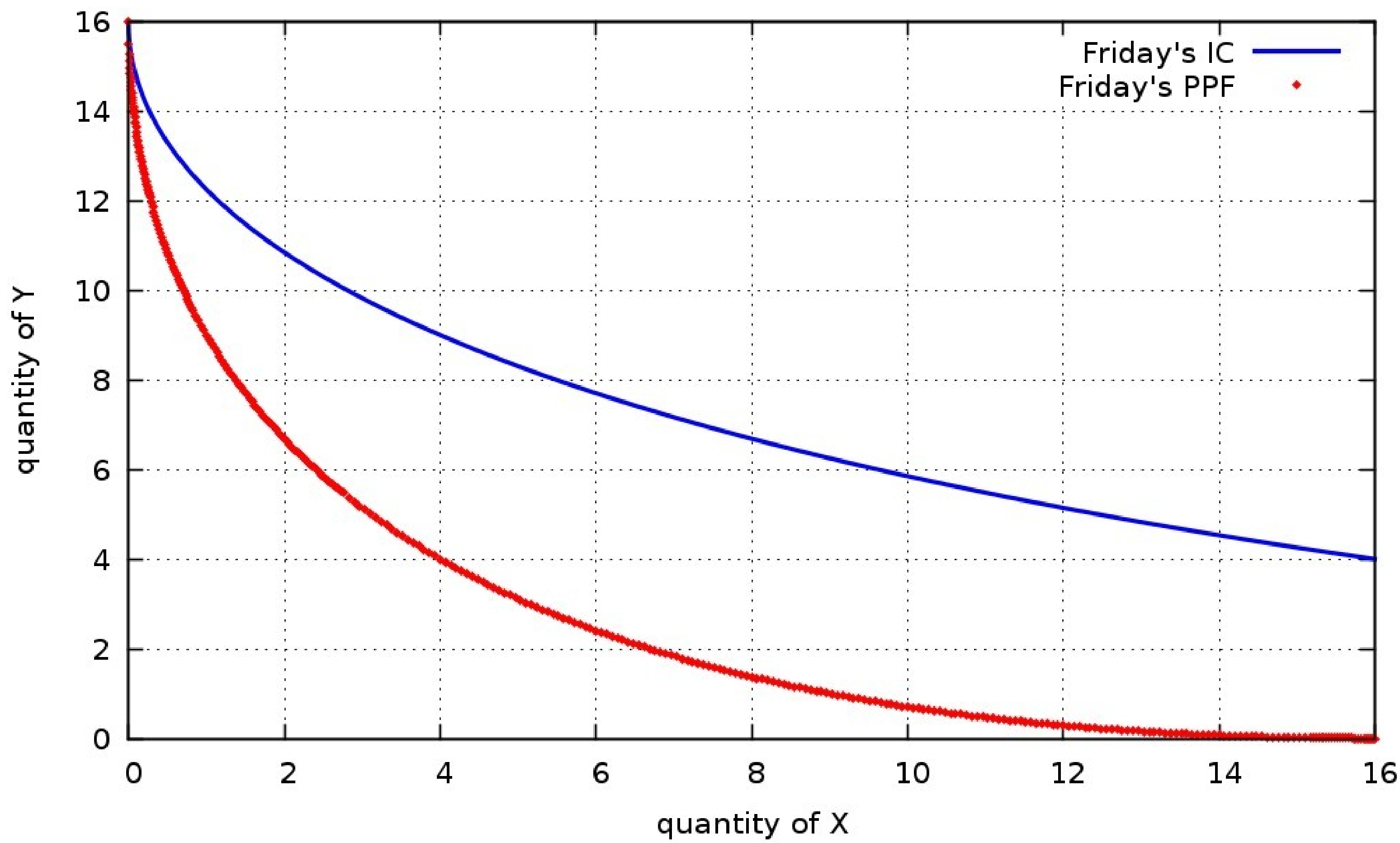
→ It is optimal for Friday to produce and consume X=0 and Y=16 because his opportunity cost of producing an "additional" unit of X is greater than what he (the consumer) is willing to pay for additional X (opp cost of X greater than marginal willingness to pay)

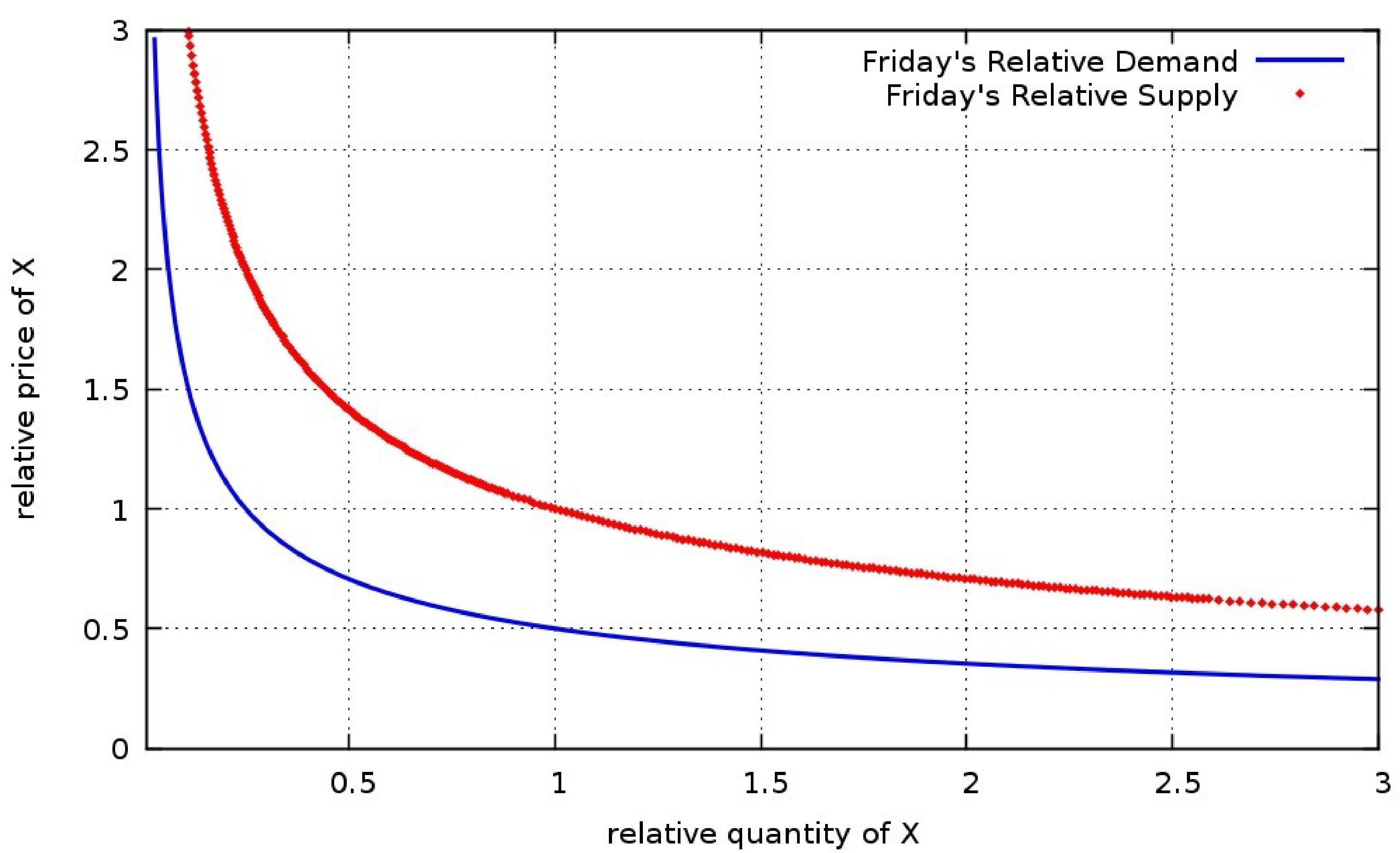


Robinson's IC
Robinson's PPF
quantity of Y
quantity of X
0
2
4
6
8
10
12
14
16



Robinson's Relative Demand
Robinson's Relative Supply
relative price of X
relative quantity of X
0
0.5
1
1.5
2
2.5
3


Friday's IC
Friday's PPF
quantity of Y
quantity of X
0
2
4
6
8
10
12
14
16


Friday's Relative Demand
Friday's Relative Supply
relative price of X
relative quantity of X
0
0.5
1
1.5
2
2.5
3