Graduate Micro
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10 Sept 2014

Problem Set

Part One – The problems in this section are designed to help you understand my "Robinson Crusoe Model" (PDF).

In this section, we will look at how the production function's shape – i.e. whether it exhibits diminishing, constant or increasing marginal returns – affects the shape of the production possibilities frontier (PPF). And because the relative supply curve is derived from the PPF, we will also look at how the production function's shape affects the shape of the relative supply curve.

We will also look at how the utility function's shape – here[1], the substitutability or complementarity between the two goods – affects the curvature of the indifference curve. And because the relative demand curve is derived from the indifference curve, we will also look at how the utility function's shape affects the relative demand curve's shape.

Specifically, we will look at two cases. In Robinson's case, the production functions will exhibit *diminishing marginal returns* and the utility function will exhibit a high degree of *complementarity* between the two goods. In Friday's case, the production functions will exhibit *increasing marginal returns* and the utility function will exhibit a high degree of *substitutability* between the two goods.

Robinson's Case

To explore the effect that *diminishing marginal returns* has on the shape of the PPF and the relative supply curve, we will assume that Robinson's total production of X and Y equal the square root of the labor allocated to the production of each:

$$X_{RC} = \sqrt{L_{RC,X}}$$

$$Y_{RC} = \sqrt{L_{RC,Y}}$$

We will also assume that Robinson has 128 units of labor:

$$128 = L_{RC,X} + L_{RC,Y}$$

To explore the effect that a high degree of *complementarity* has on the shape of the indifference curve and relative demand curve, we will assume that Robinson's utility function is given by:

$$U_{RC} = \frac{1}{\frac{1}{X_{RC}} + \frac{1}{Y_{RC}}}$$

Finally, at his utility maximum, Robinson obtains four units of satisfaction:

$$U^*_{RC} = 4$$

Friday's Case

To explore the effect that *increasing marginal returns* has on the shape of the PPF and the relative supply curve, we will assume that Friday's total production of X and Y equal the square of the labor allocated to the production of each.

$$X_{Fri} = L^2_{Fri,X}$$

$$Y_{Fri} = L^2_{Fri,Y}$$

We will also assume that Friday has 4 units of labor:

$$4 = L_{Fri,X} + L_{Fri,Y}$$

To explore the effect that a high degree of *substitutability* has on the shape of the indifference curve and relative demand curve, we will assume that Friday's utility function is given by:

$$U_{Fri} = \left(\sqrt{X_{Fri}} + 2\sqrt{Y_{Fri}}\right)^2$$

Finally, at his utility maximum, Friday obtains 64 units of satisfaction:

$$U^*_{Fri} = 64$$

[1] Because this is a "Robinson Crusoe model," we will only explore the substitution effect, not the income effect.

Use the assumptions in Robinson's case to complete the following tasks. Then complete them using the assumptions in Friday's case.

1. Derive the PPF equation from the production functions and the labor constraint. In other words, find the equation that expresses the quantity of Y produced for each quantity of X produced.
2. Holding the level of utility constant at the maximum value, U^*, derive the indifference curve's equation from the utility function. In other words, find the equation that expresses the quantity of Y consumed for each quantity of X consumed while holding utility constant at U^*.
3. Plot the PPF and the indifference curve.
   - At what quantity of X and Y does Robinson maximize his utility? At what quantity does Friday?
   - How do our assumptions about marginal returns affect the shape of the PPF? In other words, why is Robinson's PPF concave to the origin and why is Friday's PPF convex to the origin?
   - How do our assumptions about complementarity/substitutability affect the shape of the indifference curve? In other words, why is Friday's indifference curve "flatter" than Robinson's?

The opportunity cost of producing X is the additional cost (in terms of foregone Y) incurred by the production of an additional unit of X. Because opportunity cost is the change in Y produced divided by the change in X produced, the opportunity cost of producing X is the slope of the PPF, so we can calculate it by taking the derivative of the PPF equation.

Similarly, the marginal rate of substitution (MRS) is the reduction in Y necessary to hold utility constant when an additional unit of X is consumed. Being the change in Y divided by the change in X, the MRS is the slope of the indifference curve and we can calculate it by taking the derivative of the indifference curve.

We also use the derivatives to develop the relative supply and relative demand curves. The relative supply curve expresses the relationship between the opportunity cost of producing X and the relative quantity of X supplied. The relative demand curve expresses the relationship between the MRS and the relative quantity of X demanded. Consequently, we use the derivative of the PPF to derive relative supply and we use the derivative of the indifference curve to derive relative demand.

4. Derive relative supply:
   - Take the derivative of the PPF and express it in terms of the relative quantity of X.
   - Alternatively, create a table of the quantities of X and Y produced. Then measure the opportunity cost of X by calculating the PPF's slope at each value of X produced. Finally, list the relative supply schedule by creating a table of the opportunity cost values and the corresponding relative quantities of X.
5. Derive relative demand:
   - Take the derivative of the indifference curve and express it in terms of the relative quantity of X.
   - Alternatively, create a table of the quantities of X and Y consumed at U^*. Then measure the MRS by calculating the indifference curve's slope at each value of X produced. Finally, list the relative demand schedule by creating a table of the MRS values and the corresponding relative quantities of X.
6. Plot the relative supply and relative demand curves.
   - How do our assumptions about marginal returns affect the shape of the relative supply curve?
     - Why does Robinson's relative supply curve slope upward?
     - Why does Friday's relative supply curve slope downward?
   - How do our assumptions about complementarity/substitutability affect the shape of the relative demand curve? In other words, why is Friday's relative demand curve more elastic than Robinson's?
   - What is Robinson's equilibrium relative quantity of X? What is his equilibrium relative price of X?
   - Using what you know about opportunity cost and the marginal rate of substitution, explain why:
     - Friday's equilibrium relative quantity of X is zero
     - Friday's equilibrium does **not** occur where the relative supply and relative demand curves cross.

Part Two – Suppose that the world market price of soybeans is $4 per bushel, but the US government imposes a tariff on imported soybeans of $1 per bushel. Suppose further that American consumers purchase 2.5 billion bushels of soybeans from American farmers and another 1.5 billion bushels of soybeans from foreign producers at the domestic price of soybeans.

Suppose that it has been estimated that if the US government were to repeal the tariff on soybeans, American farmers would only sell 2 billion bushels of soybeans to American consumers. Lastly, suppose that it has also been estimated that the American demand curve for soybeans is linear and the price elasticity of demand is equal to -1.25 at the domestic price of soybeans.

1. What is the domestic price of soybeans?
2. What is the total number of bushels of soybeans that American consumers would buy at the world market price?
3. If the American soybean market supply curve is linear, then what is the price elasticity of American soybean supply at the world market price?
4. If the American soybean market supply curve is linear, then what is the price elasticity of American soybean supply at the domestic market price?
5. Graph the American supply and demand curves. On your graph:
   - show the quantity of soybeans demanded by American consumers at the domestic market price and at the world market price,
   - show the quantity of soybeans supplied by American farmers at the domestic market price and at the world market price and
   - show the intercepts of the supply and demand curves.

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Part Three – Consider an industry in which there are a large number of potential firms. Each firm in the industry has the same production process and produces output using capital and labor. Assume that capital and labor both exhibit diminishing marginal returns, so that capital can be substituted for labor in the production process (and vice versa), but capital and labor are not perfect substitutes.

Assume further that each firm is too small to affect the market wage rate for labor and that each firm is too small to affect the market rental rate on capital. Finally, assume that when the firm uses capital, it discharges pollutants into the local river.

Because the town's drinking water comes from the local river, the townspeople are concerned about water quality and ask the town council to force the firms to discharge less pollution into the local river.

One councilman responds by proposing a tax per unit of pollution that is discharged into the river.

1. Suppose that each firm in the industry minimizes the cost of producing a given level of output.
   - How would such a tax affect the relative wage rate?
   - How would such a tax affect the optimal combination of capital and labor that each firm uses to produce output?
   - Illustrate your answer to the previous question with isoquants and isocosts.
2. Now suppose that each firm in the industry is free to choose its optimal level of output.
   - How would such a tax affect a firm's marginal cost curve?
   - How would such a tax affect a firm's average cost curve?
   - How would such a tax affect the economic profit of firms in the industry?
3. If firms can freely enter and exit the industry, then:
   - How would such a tax affect the market supply curve in the industry?
   - How would such a tax affect the market equilibrium price of output?
4. Explain how such a tax would reduce the amount of pollution discharged into the river.