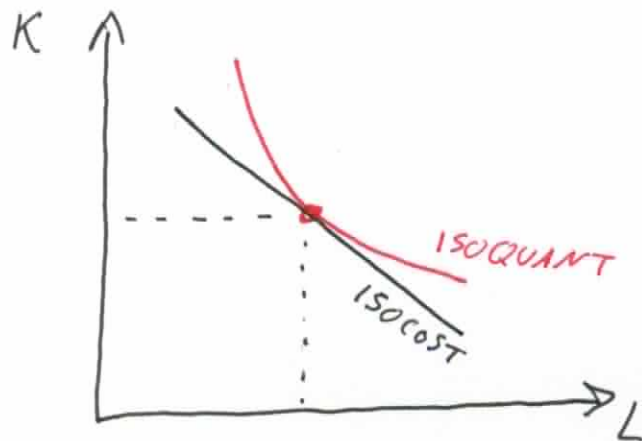


# STOLPER-SAMUELSON EXAMPLE

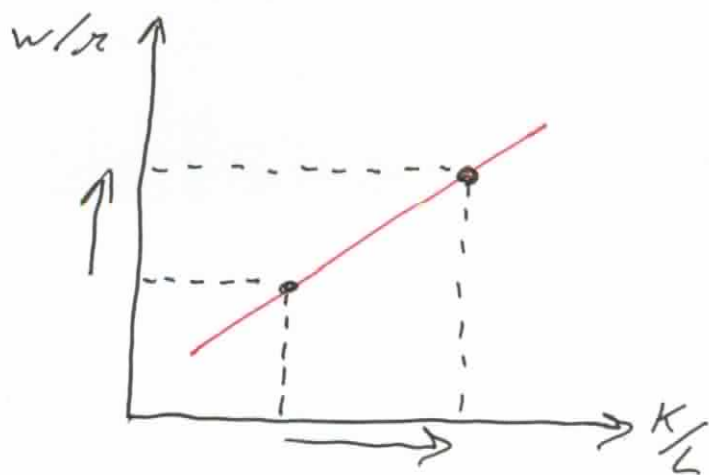
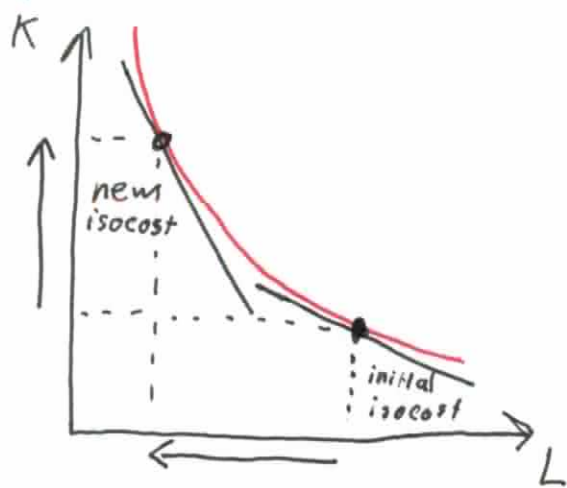
P.1

Recall that a cost minimizing firm selects the optimal amounts of capital,  $K$  and labor,  $L$  at the point where the isoquant is tangent to the isocost

$$\text{slope of isocost } \frac{w}{r} = \frac{MPL}{MPK} \text{ slope of isoquant}$$



If the relative wage rises, the firm's capital-labor ratio would rise



Suppose that there are two sectors  
Cloth and Manufactures

pg 2

Production Function  
in Cloth sector

$$C = K_c^\alpha L_c^{1-\alpha}$$

$$\frac{MPL_c}{MPK_c} = \frac{(1-\alpha) K_c^\alpha L_c^{-\alpha}}{\alpha K_c^{\alpha-1} L_c^{1-\alpha}}$$

$$\frac{MPL_c}{MPK_c} = \frac{1-\alpha}{\alpha} \cdot \frac{K_c}{L_c}$$

slope of isoquant  
in the cloth sector

$$\frac{w}{r} = \frac{1-\alpha}{\alpha} \cdot \frac{K_c}{L_c}$$

OR

$$\frac{K_c}{L_c} = \frac{\alpha}{1-\alpha} \cdot \frac{w}{r}$$

Production Function  
in Manuf. sector

$$M = K_M^{1-\alpha} L_M^\alpha$$

$$\frac{MPL_M}{MPK_M} = \frac{\alpha K_M^{1-\alpha} L_M^{\alpha-1}}{(1-\alpha) K_M^{-\alpha} L_M^\alpha}$$

$$\frac{MPL_M}{MPK_M} = \frac{\alpha}{1-\alpha} \cdot \frac{K_M}{L_M}$$

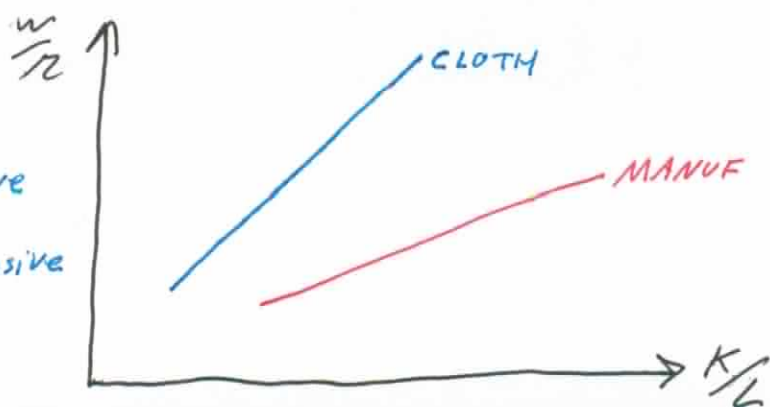
slope of isoquant  
in the manufactures sector

$$\frac{w}{r} = \frac{\alpha}{1-\alpha} \cdot \frac{K_M}{L_M}$$

OR

$$\frac{K_M}{L_M} = \frac{1-\alpha}{\alpha} \cdot \frac{w}{r}$$

IF  $\alpha = \frac{1}{3}$ , then  
cloth is labor-intensive  
manuf. is capital-intensive



Next recall that at an optimum  
the relative price of cloth must  
be equal to the opportunity cost  
of producing cloth

P. 3

$$\text{rel price of cloth} \quad \frac{P_c}{P_M} = \frac{MPL_M}{MPL_c} = \frac{MPK_M}{MPK_c} \quad \text{opp cost of producing cloth}$$

$$\frac{P_c}{P_M} = \frac{\alpha K_M^{1-\alpha} L_M^{\alpha-1}}{(1-\alpha) K_c^\alpha L_c^{-\alpha}} = \frac{(1-\alpha) K_M^{-\alpha} L_M^\alpha}{\alpha K_c^{\alpha-1} L_c^{1-\alpha}}$$

$$\frac{P_c}{P_M} = \frac{\alpha}{1-\alpha} \left(\frac{K_M}{L_M}\right)^{1-\alpha} \left(\frac{K_c}{L_c}\right)^{-\alpha} = \frac{1-\alpha}{\alpha} \left(\frac{K_M}{L_M}\right)^{-\alpha} \left(\frac{K_c}{L_c}\right)^{1-\alpha}$$

$$\left(\frac{K_M}{L_M}\right)^\alpha \left(\frac{K_c}{L_c}\right)^\alpha \frac{P_c}{P_M} = \frac{\alpha}{1-\alpha} \cdot \frac{K_M}{L_M} = \frac{1-\alpha}{\alpha} \cdot \frac{K_c}{L_c} = \frac{w}{r}$$

$$\frac{w}{r} = \frac{P_c}{P_M} \cdot \left(\frac{K_c}{L_c}\right)^\alpha \left(\frac{K_M}{L_M}\right)^\alpha$$

$$\text{but } \frac{K_c}{L_c} = \frac{\alpha}{1-\alpha} \cdot \frac{w}{r} \quad \text{and} \quad \frac{K_M}{L_M} = \frac{1-\alpha}{\alpha} \cdot \frac{w}{r}$$

$$\frac{w}{r} = \frac{p_c}{p_m} \cdot \left( \frac{\alpha}{1-\alpha} \cdot \frac{w}{r} \right)^\alpha \left( \frac{1-\alpha}{\alpha} \cdot \frac{w}{r} \right)^\alpha$$

P. 4

$$\frac{w}{r} = \frac{p_c}{p_m} \cdot \left( \frac{w}{r} \right)^{2\alpha} \cdot \underbrace{\left( \frac{\alpha}{1-\alpha} \right)^\alpha \cdot \left( \frac{1-\alpha}{\alpha} \right)^\alpha}_{\text{one}}$$

$$\frac{p_c}{p_m} = \left( \frac{w}{r} \right)^{1-2\alpha}$$

If  $\alpha = \frac{1}{3}$  then cloth is labor-intensive and manufactures are capital-intensive

So if  $\alpha = \frac{1}{3}$  then an increase in the price of the labor-intensive good (cloth) will raise the relative wage

Also note that if  $\alpha = \frac{2}{3}$  then cloth would be capital intensive and  $1-2\alpha < 0$ , so an increase in the price of cloth would reduce the relative wage.